The Kelvin, Kepler and Quantizer problem: Crystalline optimal forms and meta-stable disordered forms

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ABSTRACT

Partitioning space into cells is central to many fields of science and technology, as well as to resource distribution problems in economics and telecommunication. The nature of such cellular partitions is often defined by optimization with respect to certain properties, such as interface area in the Kelvin problem, packing density in the Kepler problem, or cell centrality as in the Quantizer problem. In all known cases, the optimal solutions are crystalline configurations with long range order. Amorphous disordered structures are generally considered to be intermittent metastable states that prevent the system from attaining the optimal ordered structures. To date, no optimization problem has been identified where the optimal solution is a disordered configuration. In this talk, we will use Lloyds algorithm to show that a stable disordered state exists in the three-dimensional Quantizer problem, despite the existence of lower-energy crystalline configurations. Akin to a thermodynamic phase, this state is universal; i.e., irrespective of the level and type of disorder in the initial configurations, we find a convergence to the same amorphous state, representing configurations characterized by the same structure factor and energy distributions. This highly degenerate state is characterised by an anomalous suppression of long-wavelength density fluctuations, known as hyperuniformity.