The flow of foam through vein-like geometries

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ABSTRACT

Varicose veins are enlarged and swollen veins which arise due to the reduction in blood flow caused by damaged valves. Increased pressure within the vein causes discomfort and can lead to further medical complications. Of the treatments available, the least invasive method consists of the injection of a surfactant-laden foam directly into the affected vein [1]. The foam displaces the stagnant blood and the surfactant treats the walls of the channel, causing the vein to collapse and eventually dissolve.

Foam is used for several reasons: its yield stress prevents gravity override and aids the displacement of blood, and less surfactant is required to coat the walls compared to the injection of pure surfactant. The yield stress can be empirically estimated from the foams properties. We model the foam as a Bingham fluid.

We derive an exact solution for the steady pressure-driven flow of a Bingham fluid in a curved 2D channel, giving expressions for the velocity and stress, concentrating particularly on the yielded and un-yielded regions in the channel [2]. The velocity profile is used to validate FEM simulations, which are then applied to other non-straight vein-like geometries.

In particular, we simulate the flow from a straight to a curved channel and trace the developing velocity profile. We determine the proportion of fluid in the un-yielded (plug) region for different values of the Bingham number and the channel curvature.

The sclerotherapy treatment will work better if the plug region in the foam is wider. We quantify how an increase in channel curvature decreases the area of the plug region, and therefore reduces the effectiveness of the treatment. So during treatment a patient's leg should be kept as straight as possible.

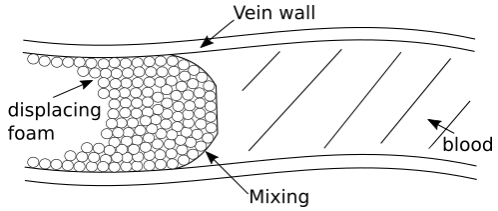

Figure 1: An illustration of the use of foam in Varicose vein sclerotherapy.

[1] R.Nael and S.Rathburn. Effectiveness of foam sclerotherapy for the treatment of varicose veins, *Vascular Medicine*, 15(1):2732, 2010.

[2] T. Roberts and S. Cox. An analytic velocity profile for pressure-driven flow of a Bingham fluid in a curved channel, arXiv.org, 2019.