

# Viscous froth model applied to the motion of a three bubble system

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## ABSTRACT

The viscous froth model (VFM) is used to predict the rheological behaviour of a two-dimensional liquid-foam system comprised of three bubbles moving between two glass plates (a). The VFM incorporates three physical phenomena: the viscous drag force, the pressure difference across films, and the surface tension acting along the films, converting any mismatch between the pressure and the film curvature, to film motion [1]. Using VFM, it was proven that in the so-called infinite staircase structure (b), the system does not undergo topological transformations, for any arbitrarily high driving velocity, meaning that the bubbles flow out of the transport channel in the same order in which they entered it [2]. In contrast, in a simple staircase, the so-called bubble lens system (c), for higher driving velocities the system undergoes topological transformations [1]. To investigate under which physical and geometric conditions those topological changes take place, a three bubble symmetric case is studied, to understand the system behaviour as it is set in motion to higher migration velocities. This structure is set in motion out of equilibrium, by imposing back pressures. For low imposed back pressures, perturbation solutions are obtained, whilst numerical solutions are obtained for higher pressures, leading to steady-state solutions. For a sufficiently high imposed back pressure, i.e. a critical back pressure, the systems undergo topological transformations, or else eventually the steady-state solutions break down at a saddle-node bifurcation, where new solution branches appear. To determine the systems' final shapes, and how topological transformations occur, unsteady-state simulations are realized for pressures higher than critical.

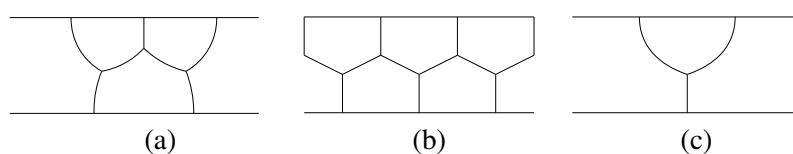


Figure 1: (a) Three bubbles symmetric case. (b) Staircase. (c) single lens.

[1] Green, T. E., et. al. (2006). Viscous froth lens. *Physical Review E*, vol. 74(5), pp 051403.

[2] Cox, S. J., et. al. (2009). The viscous froth model: steady states and the high-velocity limit. *Proceedings of the Royal Society A*, vol. 465(2108), pp 2391–2405.